

COMPACT AND “COMPACT” OPERATORS ON STANDARD HILBERT MODULES OVER C^* -ALGEBRAS

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ABSTRACT. We construct a topology on the standard Hilbert module $H_{\mathcal{A}}$ over a unital C^* -algebra and topology on $H_{\mathcal{A}}^{\#}$ (the extension of the module $H_{\mathcal{A}}$ by the algebra \mathcal{A}^{**}) such that any “compact” operator (i.e. any operator in the norm closure of the linear span of the operators of the form $z \mapsto x \langle y, z \rangle$, $x, y \in H_{\mathcal{A}}$ (or $x, y \in H_{\mathcal{A}}^{\#}$)) maps bounded sets into totally bounded sets.

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