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ON AN ELASTO-ACOUSTIC TRANSMISSION PROBLEM IN ANISOTROPIC, INHOMOGENEOUS MEDIA

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Abstract. We consider a coupled system describing the interaction between acoustic and elastic regions, where the coupling occurs not via material properties but through an interaction on an interface separating the two regimes. Evolutionary well-posedness in the sense of Hadamard well-posedness supplemented by causal dependence is shown for a natural choice of generalized interface conditions. The results are obtained in a real Hilbert space setting incurring no regularity constraints on the boundary and almost none on the interface of the underlying regions.

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