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ON TENSORS OF FACTORIZABLE QUANTUM CHANNELS WITH THE COMPLETELY DEPOLARIZING CHANNEL

YUKI UEDA

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ABSTRACT. In this paper, we obtain results for factorizability of quantum channels. Firstly, we prove that if a tensor $T \otimes S_k$ of a quantum channel T on $M_n(\mathbb{C})$ with the completely depolarizing channel S_k is written as a convex combination of automorphisms on the matrix algebra $M_n(\mathbb{C}) \otimes M_k(\mathbb{C})$ with rational coefficients, then the quantum channel T has an exact factorization through some matrix algebra with the normalized trace. Next, we prove that if a quantum channel has an exact factorization through a finite dimensional von Neumann algebra with a convex combination of normal faithful tracial states with rational coefficients, then it also has an exact factorization through some matrix algebra with the normalized trace.

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Department of Mathematics, Hokkaido University, Kita 10, Nishi 8, Kita-Ku, Sapporo, Hokkaido, 060-0810, Japan.

E-mail address: yuuki1114@math.sci.hokudai.ac.jp