

QUANTUM GROUPS, FROM A FUNCTIONAL ANALYSIS PERSPECTIVE

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Dedicated to the memory of Stefan Banach

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ABSTRACT. It is well-known that any compact Lie group appears as closed subgroup of a unitary group, $G \subset U_N$. The unitary group U_N has a free analogue U_N^+ , and the study of the closed quantum subgroups $G \subset U_N^+$ is a problem of general interest. We review here the basic tools for dealing with such quantum groups, with all the needed preliminaries included, and we discuss as well a number of more advanced topics.

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