

## THE STRUCTURE OF FRACTIONAL SPACES GENERATED BY A TWO-DIMENSIONAL NEUTRON TRANSPORT OPERATOR AND ITS APPLICATIONS

ALLABEREN ASHYRALYEV<sup>1\*</sup> and ABDULGAFUR TASKIN<sup>2</sup>

Communicated by A. Kaminska

ABSTRACT. In this study, the structure of fractional spaces generated by the two-dimensional neutron transport operator  $A$  defined by formula  $Au = \omega_1 \frac{\partial u}{\partial x} + \omega_2 \frac{\partial u}{\partial y}$  is investigated. The positivity of  $A$  in  $C(\mathbb{R}^2)$  and  $L_p(\mathbb{R}^2)$ ,  $1 \leq p < \infty$ , is established. It is established that, for any  $0 < \alpha < 1$  and  $1 \leq p < \infty$ , the norms of spaces  $E_{\alpha,p}(L_p(\mathbb{R}^2), A)$  and  $E_\alpha(C(\mathbb{R}^2), A)$ ,  $W_p^\alpha(\mathbb{R}^2)$  and  $C^\alpha(\mathbb{R}^2)$  are equivalent, respectively. The positivity of the neutron transport operator in Hölder space  $C^\alpha(\mathbb{R}^2)$  and Slobodeckij space  $W_p^\alpha(\mathbb{R}^2)$  is proved. In applications, theorems on the stability of Cauchy problem for the neutron transport equation in Hölder and Slobodeckij spaces are provided.

### REFERENCES

1. S. Agmon and L. Nirenberg, *Properties of solutions of ordinary differential equations in Banach spaces*, Comm. Pure Appl. Math. **16** (1963), 121–239.
2. A. Ashyralyev, *A survey of results in the theory of fractional spaces generated by positive operators*, TWMS J. Pure Appl. Math. **6** (2015), no. 2, 129–157.
3. A. Ashyralyev and S. Akturk, *Positivity of a one-dimensional difference operator in the half-line and its applications*, Appl. Comput. Math. **14** (2015), no. 2, 204–220.
4. A. Ashyralyev and A. Taskin, *Structure of fractional spaces generated by the two dimensional neutron transport operator*, AIP Conf. Proc. **1759** (2016), 661–665.

---

Copyright 2018 by the Tusi Mathematical Research Group.

Date: Received: Nov. 12, 2017; Accepted: Apr. 18, 2018.

\*Corresponding author

$\diamond$  Advance publication – final volume, issue, and page numbers to be assigned.

2010 *Mathematics Subject Classification*. Primary 47B65; Secondary 35A35, 35K30, 34B27.

*Key words and phrases*. Neutron transport operator, fractional space, Slobodeckij space, positive operator.

5. A. Ashyralyev and F. S. Tetikoglu, *A note on fractional spaces generated by the positive operator with periodic conditions and applications*, Bound. Value Probl. **2015**, 2015:31, 17 pp.
6. A. Ashyralyev and P. E. Sobolevskii, *Well-posedness of parabolic difference equations*, Operator Theory: Advances and Applications vol. 69, Birkhäuser, Verlag, Basel, Boston, Berlin, 1994.
7. A. Ashyralyev, N. Nalbant, and Y. Sozen, *Structure of fractional spaces generated by second order difference operators*, J. Franklin Inst. **351** (2014), no. 2, 713–731.
8. H. O. Fattorini, *Second order linear differential equations in Banach spaces*, Elsevier Science Publishing Company, North-Holland, Amsterdam, 1985.
9. S. G. Krein, *Linear differential equations in Banach space*, Translated from the Russian by J. M. Danskin. Translations of Mathematical Monographs, Vol. 29. American Mathematical Society, Providence, R.I., 1971.
10. V. I. Lebedeva and P. E. Sobolevskii, *Spectral properties of the transfer operator with constant coefficients in  $L_p(R^n)$  ( $1 \leq p < \infty$ ) spaces*, Voronezh. Gosud. Univ. 1983, 54 pages. Deposited VINITI, 02.06.1983, no. 2958-83, (Russian) 1983.
11. V. I. Lebedeva, *Spectral properties of the transfer operator of neutron in  $C(\Omega, C(R^n))$  spaces*, Qualitative and Approximate Methods for Solving Operator Equations, Yaroslavl, (Russian) **9** (1984), 44–51.
12. E. Lewis and W. Miller *Computational methods of neutron transport*, American Nuclear Society, USA, 1993.
13. G. I. Marchuk and V. I. Lebedev, *Numerical methods in the theory of neutron transport*, Taylor and Francis, USA, 1986.
14. M. Mokhtar-Kharroubi, *Mathematical topics in neutron transport theory, New aspects*, World Scientific, Singapore and River Edge, N.J., 1997.
15. V. Shakhmurov and H. Musaev, *Maximal regular convolution-differential equations in weighted Besov spaces*, Appl. Comput. Math. **16** (2017), no. 2, 190–200.
16. P. E. Sobolevskii, *Some properties of the solutions of differential equations in fractional spaces*, Trudy Nauchn.-Issled. Inst. Mat. Voronezh. Gos. Univ., (Russian) **74** (1975), 68–76.
17. H. Triebel, *Interpolation theory, function spaces, differential operators*, North-Holland Mathematical Library, 18. North-Holland Publishing Co., Amsterdam-New York, 1978.
18. V. I. Zhukova, *Spectral properties of the transfer operator*, Trudy Vsesoyuznoy Nauchno-Prakticheskoy Konferensii, Chita **5** (2000), no. 1, 170–174.
19. V. I. Zhukova and L. N. Gamolya, *Investigation of spectral properties of the transfer operator*, Dalnovostochniy Matematicheskiy Zhurnal, (Russian) **5** (2004), no. 1, 158–164.

<sup>1</sup>DEPARTMENT OF MATHEMATICS, NEAR EAST UNIVERSITY, NICOSIA, TRNC, MERSIN 10, TURKEY;

PEOPLES' FRIENDSHIP UNIVERSITY OF RUSSIA (RUDN UNIVERSITY), ULMIKLUKO MAKLAYA 6, MOSCOW 117198, RUSSIA;

INSTITUTE OF MATHEMATICS AND MATHEMATICAL MODELING, 050010, ALMATY, KAZAKHSTAN.

*E-mail address:* [allaberen.ashyralyev@neu.edu.tr](mailto:allaberen.ashyralyev@neu.edu.tr), [aashyr@yahoo.com](mailto:aashyr@yahoo.com)

<sup>2</sup>DEPARTMENT OF MATHEMATICS, PRIVATE SOYAK BAHCESEHIR SCIENCE AND TECHNOLOGY COLLEGE, UMRANIYE, ISTANBUL, TURKEY.

*E-mail address:* [abdulgafur.taskin@bahcesehir.k12.tr](mailto:abdulgafur.taskin@bahcesehir.k12.tr), [gafurtaskin@hotmail.com](mailto:gafurtaskin@hotmail.com)