

## $L^p$ -HARDY–RELLICH AND UNCERTAINTY PRINCIPLE INEQUALITIES ON THE SPHERE

ABIMBOLA ABOLARINWA<sup>1\*</sup> and TIMOTHY APATA<sup>2</sup>

Communicated by D. M. Pellegrino

**ABSTRACT.** In this paper, we study the Hardy–Rellich type inequalities and uncertainty principle on the geodesic sphere. Firstly, we derive  $L^p$ -Hardy inequalities via divergence theorem, which are in turn used to establish the  $L^p$ -Rellich inequalities. We also establish Heisenberg uncertainty principle on the sphere via the Hardy–Rellich type inequalities. The best constants appearing in the inequalities are shown to be sharp.

### REFERENCES

1. E. Berchio, L. D’Ambrosio, D. Ganguly, and G. Grillo, *Improved  $L^p$ -Poincaré inequalities on the hyperbolic space*, *Nonlinear Anal.* **157** (2017), 146–166.
2. H. Brezis and J. L. Vázquez, *Blow-up solutions of some nonlinear elliptic problems*, *Rev. Mat. Univ. Comp. Madrid* **10** (1997), no. 2, 443–469.
3. G. Carron, *Inégalités de Hardy sur les variétés riemanniennes non-compactes*, *J. Math. Pures Appl.*, **76** (1997), no. 10, 883–891.
4. I. Chavel, *Eigenvalues in Riemannian geometry*. New York: Academic Press, 1984.
5. I. Chavel, *Riemannian geometry: a modern introduction*, second edition *Cambridge Tracts in Mathematics*, 108. Cambridge University Press, Cambridge, 2006.
6. F. Dai and Y. Xu, *The Hardy-Rellich inequality and uncertainty principle inequalities on the sphere*, <http://arxiv.org/abs/1212.3887v3> [math.CA], 2014.
7. E. B. Davies and A. M. Hinz, *Explicit constants for Rellich inequalities in  $L^p(\Omega)$* , *Math. Z.* **227** (1998), no. 3, 511–523.

---

Copyright 2018 by the Tusi Mathematical Research Group.

*Date:* Received: Jan 01, 2018; Accepted: Apr. 15, 2018.

\*Corresponding author

$\diamond$  Advance publication – final volume, issue, and page numbers to be assigned.

2010 *Mathematics Subject Classification.* Primary 26D10; Secondary 46E30, 53C21.

*Key words and phrases.* Hardy inequalities, Rellich inequalities, uncertainty principle, geodesic sphere, compact manifold.

8. M. M. Fall and F. Mahmoudi, *Weighted Hardy inequality with higher dimensional singularity on the boundary*, Calc. Var. Partial Differential Equations **50** (2014), no. 3-4, 779–798.
9. G. B. Folland and A. Sitaram, *The uncertainty principle: a mathematical survey*, J. Fourier Anal. Appl. **3** (1997), no. 3, 207–233.
10. S. Gallot, D. Hulin, and J. Lafontaine, *Riemannian geometry*, Universitext. Springer-Verlag, Berlin, 1987.
11. N. Garofalo and E. Lanconelli, *Frequency functions on the Heisenberg group, the uncertainty principle and unique continuation*, Ann. Inst. Fourier (Grenoble) **40** (1990), no. 2, 313–356.
12. S. S. Goh and T. N. Goodman, *Uncertainty principles and asymptotic behavior*, Appl. Comput. Harmon. Anal. **16** (2004), no. 1, 69–89.
13. I. Kombe, *Hardy, Rellich and uncertainty principle inequalities on Carnot groups*, arXiv:math/0611850, 2006.
14. I. Kombe and M. Özaydin, *Improved Hardy and Rellich inequalities on Riemannian manifolds*, Trans. Amer. Math. Soc. **361** (2009), no. 12, 6191–6203.
15. I. Kombe and M. Özaydin, *Hardy-Poincaré, Rellich and uncertainty principle inequalities on Riemannian manifolds*, Trans. Amer. Math. Soc. **365** (2013), no. 10, 5035–5050.
16. I. Kombe and A. Yener, *Weighted Hardy and Rellich type inequalities on Riemannian manifolds*, Math. Nachr. **289** (2016), no. 8-9, 994–1004.
17. P. Lindqvist, *On the equation  $\operatorname{div}(|\nabla u|^{p-2}\nabla u) + \lambda|u|^{p-2}u = 0$* , Proc. Amer. Math. Soc. **109** (1990), no. 1, 157–164.
18. F. J. Narcowich and J. D. Ward, *Nonstationary wavelets on the  $m$ -sphere for scattered data*, Appl. Comput. Harmon. Anal. **3** (1996), no. 4, 324–336.
19. A. Sitaram, M. Sundari, and S. Thangavelu, *Uncertainty principles on certain Lie groups*, Proc. Indian Acad. Sci. Math. Sci. **105** (1995), no. 2, 135–151.
20. X. Sun and F. Pan, *Hardy type inequalities on the sphere*, J. Ineq. Appl. **2017**, Paper No. 148, 8 pp.
21. Y. Xiao, *Some Hardy inequalities on the sphere*, J. Math. Inequal. **10** (2016), no. 3, 793–805.
22. Q. Yang, *Best constants in the Hardy–Rellich type inequalities on the Heisenberg group*, J. Math. Anal. Appl. **342** (2008), no. 1, 423–431.
23. Q. Yang, D. Su, and Y. Kong, *Hardy inequalities on Riemannian manifolds with negative curvature*, Commun. Contemp. Math. **16** (2014), no. 2, 1350043, 24 pp.

<sup>1</sup>DEPARTMENT OF PHYSICAL SCIENCES LANDMARK UNIVERSITY, P. M. B. 1001, OMU-ARAN, KWARA STATE, NIGERIA.

*E-mail address:* [A.Abolarinwa1@gmail.com](mailto:A.Abolarinwa1@gmail.com)

<sup>2</sup>DEPARTMENT OF MATHEMATICS AND STATISTICS, OSUN STATE COLLEGE OF TECHNOLOGY, P. M. B. 1011, ESA-OKE, OSUN STATE, NIGERIA.

*E-mail address:* [rocktm24@gmail.com](mailto:rocktm24@gmail.com)