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NORM ESTIMATES FOR RESOLVENTS OF LINEAR OPERATORS IN A BANACH SPACE AND SPECTRAL VARIATIONS

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ABSTRACT. Let P_t $(a \le t \le b)$ be a function whose values are projections in a Banach space. The paper is devoted to bounded linear operators A admitting the representation

$$A = \int_a^b \phi(t) dP_t + V,$$

where $\phi(t)$ is a scalar function and V is a compact quasi-nilpotent operator such that $P_t V P_t = V P_t$ $(a \le t \le b)$. We obtain norm estimates for the resolvent of A and a bound for the spectral variation of A. In addition, the representation for the resolvents of the considered operators is established via multiplicative operator integrals. That representation can be considered as a generalization of the representation for the resolvent of a normal operator in a Hilbert space. It is also shown that the considered operators are Kreiss-bounded. Applications to integral operators in L^p are also discussed. In particular, bounds for the upper and lower spectral radius of integral operators are derived.

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