

CHARACTERIZING PROJECTIONS AMONG POSITIVE OPERATORS IN THE UNIT SPHERE

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ABSTRACT. Let E and P be subsets of a Banach space X , and let us define the unit sphere around E in P as the set

$$Sph(E; P) := \{x \in P : \|x - b\| = 1 \text{ for all } b \in E\}.$$

Given a C^* -algebra A and a subset $E \subset A$, we shall write $Sph^+(E)$ or $Sph_A^+(E)$ for the set $Sph(E; S(A^+))$, where $S(A^+)$ denotes the unit sphere of A^+ . We prove that, for every complex Hilbert space H , the following statements are equivalent for every positive element a in the unit sphere of $B(H)$:

- (a) a is a projection;
- (b) $Sph_{B(H)}^+ \left(Sph_{B(H)}^+ (\{a\}) \right) = \{a\}$.

We also prove that the equivalence remains true when $B(H)$ is replaced with an atomic von Neumann algebra or with $K(H_2)$, where H_2 is an infinite-dimensional and separable complex Hilbert space.

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