Adv. Oper. Theory ⋄
https://doi.org/10.15352/aot.1712-1275
ISSN: 2538-225X (electronic)
https://projecteuclid.org/aot

CONVOLUTION DOMINATED OPERATORS ON COMPACT EXTENSIONS OF ABELIAN GROUPS

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Communicated by L. Turowska

Abstract. If G is a locally compact group, $CD(G)$ the algebra of convolution dominated operators on $L^2(G)$, then an important question is: Is $\mathbb{C}1 + CD(G)$ (or $CD(G)$ if G is discrete) inverse-closed in the algebra of bounded operators on $L^2(G)$?

In this note we answer this question in the affirmative, provided G is such that one of the following properties is satisfied.

(1) There is a discrete, rigidly symmetric, and amenable subgroup $H \subset G$ and a (measurable) relatively compact neighbourhood of the identity U, invariant under conjugation by elements of H, such that $\{hU : h \in H\}$ is a partition of G.
(2) The commutator subgroup of G is relatively compact. (If G is connected, this just means that G is an IN group.)

All known examples where $CD(G)$ is inverse-closed in $B(L^2(G))$ are covered by this.

References

1. A. G. Baskakov, *Abstract harmonic analysis and asymptotic estimates for elements of inverse matrices*, (Russian); translated from Mat. Zametki **52** (1992), no. 2, 17–26, 155 Math. Notes **52** (1992), no. 1-2, 764–771 (1993).

Date: Received: Dec. 16, 2017; Accepted: Apr. 3, 2018.
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⋄ Advance publication – final volume, issue, and page numbers to be assigned.
2010 *Mathematics Subject Classification.* Primary 47B35; Secondary 43A20.
Key words and phrases. Convolution dominated operator, generalised L^1-algebra, symmetric locally compact group.

2. A. G. Baskakov, *Asymptotic estimates for elements of matrices of inverse operators, and harmonic analysis*, (Russian); translated from Sibirsk. Mat. Zh. **38** (1997), no. 1, 14–28, i Siberian Math. J. **38** (1997), no. 1, 10–22.
3. A. G. Baskakov, *Estimates for the elements of inverse matrices, and the spectral analysis of linear operators*, (Russian); translated from Izv. Ross. Akad. Nauk Ser. Mat. **61** (1997), no. 6, 3–26 Izv. Math. **61** (1997), no. 6, 1113–1135.
4. I. Beltiţă and D. Beltiţă, *Erratum to: Inverse-closed algebras of integral operators on locally compact groups*, Ann. Henri Poincaré **16** (2015), no. 5, 1307–1309.
5. S. Bochner and R. S. Phillips, *Absolutely convergent Fourier expansions for non-commutative normed rings*, Ann. of Math. (2), **43** (1942), 409–418.
6. B. Farrell and T. Strohmer, *Inverse-closedness of a Banach algebra of integral operators on the Heisenberg group*, J. Operator Theory **64** (2010), no. 1, 189–205.
7. G. Fendler, K. Gröchenig, and M. Leinert, *Convolution-dominated operators on discrete groups*, Integral Equations Operator Theory **61** (2008), no. 4, 493–509.
8. G. Fendler, K. Gröchenig, and M. Leinert, *Convolution-dominated integral operators*, Non-commutative harmonic analysis with applications to probability II, 121–127, Banach Center Publ., 89, Polish Acad. Sci. Inst. Math., Warsaw, 2010.
9. G. Fendler and M. Leinert, *On convolution dominated operators*, Integral Equations Operator Theory **86** (2016), no. 2, 209–230.
10. I. Gohberg, M. A. Kaashoek, and H. J. Woerdeman, *The band method for positive and contractive extension problems*, J. Operator Theory **22** (1989), no. 1, 109–155.
11. F. P. Greenleaf, *Invariant means on topological groups and their applications*, Van Nostrand Mathematical Studies, No. 16, Van Nostrand Reinhold Co., New York-Toronto, Ont.-London, 1969.
12. K. Gröchenig and M. Leinert, *Wiener's lemma for twisted convolution and Gabor frames*, J. Amer. Math. Soc. **17** (2004), 1–18.
13. A. Hulanicki, *On the spectrum of convolution operators on groups with polynomial growth*, Invent. Math. **17** (1972), 135–142.
14. K. Iwasawa, *Topological groups with invariant compact neighborhoods of the identity*, Ann. of Math. (2) **54** (1951), 345–348.
15. V. G. Kurbatov, *Algebras of difference and integral operators*, Funktsional. Anal. i Prilozhen. **24** (1990), no. 2, 87–88.
16. V. G. Kurbatov, *Functional differential operators and equations*, Mathematics and its Applications, 473. Kluwer Academic Publishers, Dordrecht, 1999.
17. V. G. Kurbatov, *Some algebras of operators majorized by a convolution*, International Conference on Differential and Functional Differential Equations (Moscow, 1999). Funct. Differ. Equ. **8** (2001), no. 3-4, 323–333.
18. H. Leptin, *Verallgemeinerte L^1-Algebren*, Math. Ann. **159** (1965), 51–76.
19. H. Leptin, *Verallgemeinerte L^1-Algebren und projektive Darstellungen lokal kompakter Gruppen, I*, Invent. Math. **3** (1967), 257–281.
20. H. Leptin, *Darstellungen verallgemeinerter L^1-Algebren*, (German) Invent. Math. **5** (1968), 192–215.
21. H. Leptin and D. Poguntke, *Symmetry and non-symmetry for locally compact groups*, J. Funct. anal. **33** (1979), 119–134.
22. Q. Sun, *Wiener's lemma for infinite matrices with polynomial off-diagonal decay*, C. R. Math. Acad. Sci. Paris **340** (2005), no. 8, 567–570.
23. N. Wienerm, *Tauberian theorems*, Ann. of Math. (2) **33** (1932), no. 1, 1–100.

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