

AFFINE ACTIONS AND THE YANG–BAXTER EQUATION

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ABSTRACT. In this paper, the relations between the Yang–Baxter equation and affine actions are explored in detail. In particular, we classify the injective set-theoretic solutions of the Yang–Baxter equation in two ways: (i) by their associated affine actions of their structure groups on their derived structure groups, and (ii) by the C^* -dynamical systems obtained from their associated affine actions. On the way to our main results, several other useful results are also obtained.

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