



Adv. Oper. Theory \diamond

<https://doi.org/10.15352/aot.1802-1312>

ISSN: 2538-225X (electronic)

<https://projecteuclid.org/aot>

BANACH PARTIAL $*$ -ALGEBRAS: AN OVERVIEW

J.-P. ANTOINE ^{1*} and C. TRAPANI ²

Communicated by P. Aiena

ABSTRACT. A Banach partial $*$ -algebra is a locally convex partial $*$ -algebra whose total space is a Banach space. A Banach partial $*$ -algebra is said to be of type (B) if it possesses a generating family of multiplier spaces that are also Banach spaces. We describe the basic properties of these objects and display a number of examples, namely, L^p -like function spaces and spaces of operators on Hilbert scales or lattices. Finally we analyze the important cases of Banach quasi $*$ -algebras and CQ^* -algebras.

REFERENCES

1. J.-P. Antoine, F. Bagarello, and C. Trapani, *Topological partial $*$ -algebras: Basic properties and examples*, Rev. Math. Phys. **11** (1999), 267–302.
2. J.-P. Antoine, A. Inoue, and C. Trapani, *Partial $*$ -algebras and their operator r Realizations*, Kluwer, Dordrecht, 2002.
3. J.-P. Antoine and W. Karwowski, *Interpolation theory and refinement of nested Hilbert spaces*, J. Math. Phys. **22** (1981), 2489–2496.
4. J.-P. Antoine and C. Trapani, *Partial inner product spaces – Theory and applications*, Lecture Notes in Mathematics, vol. 1986, Springer-Verlag, Berlin, Heidelberg, 2009.
5. J.-P. Antoine and C. Trapani, *A note on Banach partial $*$ -algebras*, Mediterr. J. Math. **3** (2006), 67–86.
6. J.-P. Antoine, C. Trapani, and F. Tschinke, *Continuous $*$ -homomorphisms of Banach partial $*$ -algebras*, Mediterr. J. Math. **4** (2007), 357–373.

Copyright 2018 by the Tusi Mathematical Research Group.

Date: Received: Feb. 13, 2018; Accepted: Mar. 14, 2018.

*Corresponding author

\diamond Advance publication – final volume, issue, and page numbers to be assigned.

2010 *Mathematics Subject Classification*. Primary 08A55; Secondary 46J10, 47L60.

Key words and phrases. Partial $*$ -algebra, Banach partial $*$ -algebra, CQ^* -algebra, partial inner product space, operators on Hilbert scale.

7. F. Bagarello, A. Inoue, and C. Trapani, *Some classes of topological quasi *-algebras*, Proc. Amer. Math. Soc. **129** (2001), 2973–2980.
8. F. Bagarello and C. Trapani, *CQ*-algebras: Structure properties*, Publ. RIMS, Kyoto Univ. **32** (1996), 85–116.
9. F. Bagarello and C. Trapani, *L^p-spaces as quasi *-algebras*, J. Math. Anal. Appl. **197** (1996,) 810–824.
10. F. Bagarello and C. Trapani, *States and representations of CQ*-algebras*, Ann. Inst. H. Poincaré **61** (1994), 103–133.
11. J. Bergh and J. Löfström, *Interpolation spaces*, Springer-Verlag, Berlin, 1976.
12. G. Birkhoff, *Lattice theory*, 3rd ed., Amer. Math. Soc., Coll. Publ., Providence, RI., 1966.
13. J. J. F. Fournier and J. Stewart, *Amalgams of L^p and ℓ^q*, Bull. Amer. Math. Soc. **13** (1985), 1–21.
14. G. G. Gould, *On a class of integration spaces*, J. London Math. Soc. **34** (1959), 161–172.
15. R. Haag and D. Kastler, *An algebraic approach to quantum field theorem*, J. Math. Phys. **5** (1964), 848–861.
16. T. Kato, *Perturbation theory for linear operators*, Springer-Verlag, Berlin, 1976.
17. G. Köthe, *Topological Vector Spaces, I and II*, Springer-Verlag, Berlin, 1966, 1979.
18. G. Lassner, *Algebras of unbounded operators and quantum dynamics*, Physica A **124** (1984), 471–480.
19. G. Lassner, *Topological algebras and their applications in quantum statistics*, Wiss. Z. KMU-Leipzig, Math.-Naturwiss. R. **30** (1981), 572–595.
20. E. Nelson, *Note on non-commutative integration*, J. Funct. Anal. **15** (1974), 103–116
21. H. H. Schaefer, *Topological vector spaces*, Springer-Verlag, Berlin, 1971.
22. I. E. Segal, *A noncommutative extension of abstract integration*, Ann. Math. **57** (1953), 401–457.
23. S. Strătilă and L. Zsidó, *Lectures on von Neumann algebras*, Editura Academiei, Bucharest and Abacus Press, Tunbridge Wells, Kent, 1979.
24. C. Trapani, *Bounded elements and spectrum in Banach quasi *-algebras*, Studia Math. **172** (2006), 249–273.
25. C. Trapani, *Quasi *-algebras of operators and their applications*, Reviews Math. Phys. **7** (1995), 1303–1332.
26. C. Trapani, *States and derivations on quasi *-algebras*, J. Math. Phys. **29** (1988), 1885–1890.
27. C. Trapani and M. Fragoulopoulou, *Locally convex quasi *-algebras and their representations*, 2018 (in preparation).
28. A. C. Zaanan, *Integration*, North-Holland, Amsterdam, 1967.

¹INSTITUT DE RECHERCHE EN MATHÉMATIQUE ET PHYSIQUE, UNIVERSITÉ CATHOLIQUE DE LOUVAIN, B-1348 LOUVAIN-LA-NEUVE, BELGIUM.

E-mail address: jean-pierre.antoine@uclouvain.be

²DIPARTIMENTO DI MATEMATICA E INFORMATICA, UNIVERSITÀ DI PALERMO, I-90123 PALERMO, ITALY.

E-mail address: camillo.trapani@unipa.it