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ORTHOGONALITY OF BOUNDED LINEAR OPERATORS ON COMPLEX BANACH SPACES

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Abstract. We study Birkhoff–James orthogonality of compact linear operators on complex reflexive Banach spaces and obtain its characterization. By means of introducing new definitions, we illustrate that it is possible in the complex case, to develop a study of orthogonality of compact linear operators, analogous to the real case. Furthermore, earlier operator theoretic characterizations of Birkhoff–James orthogonality in the real case, can be obtained as simple corollaries to our present study. In fact, we obtain more than one equivalent characterizations of Birkhoff–James orthogonality of compact linear operators in the complex case, in order to distinguish the complex case from the real case.

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