

VARIANT VERSIONS OF THE LEWENT TYPE DETERMINANTAL INEQUALITY

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ABSTRACT. In this paper, we present a refinement of the Lewent determinantal inequality and show that the following inequality holds

$$\det \frac{I_{\mathcal{H}} + A_1}{I_{\mathcal{H}} - A_1} + \det \frac{I_{\mathcal{H}} + A_n}{I_{\mathcal{H}} - A_n} - \sum_{j=1}^n \lambda_j \det \left(\frac{I_{\mathcal{H}} + A_j}{I_{\mathcal{H}} - A_j} \right) \\ \geq \det \left[\left(\frac{I_{\mathcal{H}} + A_1}{I_{\mathcal{H}} - A_1} \right) \left(\frac{I_{\mathcal{H}} + A_n}{I_{\mathcal{H}} - A_n} \right) \prod_{j=1}^n \left(\frac{I_{\mathcal{H}} + A_j}{I_{\mathcal{H}} - A_j} \right)^{-\lambda_j} \right],$$

where $A_j \in \mathbb{B}(\mathcal{H})$, $0 \leq A_j < I_{\mathcal{H}}$, A_j 's are trace class operators and $A_1 \leq A_j \leq A_n$ ($j = 1, \dots, n$) and $\sum_{j=1}^n \lambda_j = 1$, $\lambda_j \geq 0$ ($j = 1, \dots, n$). In addition, we present some new versions of the Lewent type determinantal inequality.

REFERENCES

1. T. Furuta, J. Mićić Hot, J. E. Pečarić, and Y. Seo, *Mond-Pečarić method in operator inequalities*, Inequalities for bounded selfadjoint operators on a Hilbert space. Monographs in Inequalities, 1. ELEMENT, Zagreb, 2005.
2. I. Gohberg, S. Goldberg, and M. A. Kaashoek, *Classes of linear operators*, Vol. I. Operator Theory: Advances and Applications, 49. Birkhäuser Verlag, Basel, 1990.
3. R. A. Horn and C. R. Johnson, *Matrix analysis*, Corrected reprint of the 1985 original. Cambridge University Press, Cambridge, 1990.
4. L. Lewent, *Über einige ungleichungen*, Sitzungsber. Berl. Math. Ges. **7** (1908), 95–100.

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5. M. Lin, *A Lewent type determinantal inequality*, Taiwan. J. Math. **17** (2013), no. 4, 1303–1309.
6. A. Matković, J. Pečarić, and I. Perić, *A variant of Jensen's inequality of Mercer's type for operators with applications*, Linear Algebra Appl. **418** (2006), no. 2-3, 551–564.
7. A. McD. Mercer, *A variant of Jensen's inequality*, J. Ineq. Pure and Appl. Math., **4** (2003), no. 4, Article 73, 2 pp.
8. F. Mirzapour, A. Morassaei, and M. S. Moslehian, *More on operator Bellman inequality*, Quaest. Math. **37** (2014), no. 1, 9–17.
9. G. J. Murphy, *C*-algebra and operator theory*, Academic Press, Inc., Boston, MA, 1990.
10. J. Roojin, *Some refinements of discrete Jensen's inequality and some of its applications*, Nonlinear Func. Anal. Appl. **12** (2007), no. 1, 107–118.
11. F. Zhang, *Matrix theory, Basic results and techniques*, Second edition. Universitext. Springer, New York, 2011.

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