

CLOSEDNESS AND INVERTIBILITY FOR THE SUM OF TWO CLOSED OPERATORS

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ABSTRACT. We show a Kalton–Weis type theorem for the general case of noncommuting operators. More precisely, we consider sums of two possibly noncommuting linear operators defined in a Banach space such that one of the operators admits a bounded H^∞ -calculus, the resolvent of the other one satisfies some weaker boundedness condition and the commutator of their resolvents has certain decay behavior with respect to the spectral parameters. Under this consideration, we show that the sum is closed and that after a sufficiently large positive shift it becomes invertible and moreover sectorial. As an application we recover a classical result on the existence, uniqueness, and maximal L^p -regularity for solutions of the abstract linear nonautonomous parabolic problem.

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