

***T1* THEOREM FOR INHOMOGENEOUS TRIEBEL–LIZORKIN AND BESOV SPACES ON RD-SPACES AND ITS APPLICATION**

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ABSTRACT. Using Calderón’s reproducing formulas and almost orthogonal estimates, the *T1* theorem for the inhomogeneous Triebel–Lizorkin and Besov spaces on RD-spaces is obtained. As an application, new characterizations for these spaces with “half” the usual conditions of the approximate to the identity are presented.

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