

## A KAKUTANI–MACKEY-LIKE THEOREM

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**ABSTRACT.** We give a partial extension of a Kakutani–Mackey theorem for quasi-complemented vector spaces. This can be applied in the representation theory of certain complemented (non-normed) topological algebras. The existence of continuous linear maps, in the context of quasi-complemented vector spaces, is a very important issue in their study. Relative to this, we prove that every Hausdorff quasi-complemented locally convex space has continuous linear maps, under which a certain quasi-complemented locally convex space turns to be pre-Hilbert.

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