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PERTURBATION OF MINIMUM ATTAINING OPERATORS

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ABSTRACT. We prove that the minimum attaining property of a bounded linear operator on a Hilbert space H whose minimum modulus lies in the discrete spectrum, is stable under small compact perturbations. We also observe that given a bounded operator with strictly positive essential minimum modulus, the set of compact perturbations which fail to produce a minimum attaining operator is smaller than a nowhere dense set. In fact, it is a porous set in the ideal of all compact operators on H . Further, we try to extend these stability results to perturbations by all bounded linear operators with small norm and obtain subsequent results.

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