

ON THE TRUNCATED TWO-DIMENSIONAL MOMENT PROBLEM

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ABSTRACT. We study the truncated two-dimensional moment problem (with rectangular data) to find a non-negative measure $\mu(\delta)$, $\delta \in \mathfrak{B}(\mathbb{R}^2)$, such that $\int_{\mathbb{R}^2} x_1^m x_2^n d\mu = s_{m,n}$, $0 \leq m \leq M$, $0 \leq n \leq N$, where $\{s_{m,n}\}_{0 \leq m \leq M, 0 \leq n \leq N}$ is a prescribed sequence of real numbers; $M, N \in \mathbb{Z}_+$. For the cases $M = N = 1$ and $M = 1, N = 2$ explicit numerical necessary and sufficient conditions for the solvability of the moment problem are given. In the cases $M = N = 2$; $M = 2, N = 3$; $M = 3, N = 2$; $M = 3, N = 3$ some explicit numerical sufficient conditions for the solvability are obtained. In all the cases some solutions (not necessarily atomic) of the moment problem can be constructed.

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