

## SINGULAR RIESZ MEASURES ON SYMMETRIC CONES

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Communicated by G. Olafsson

ABSTRACT. A fundamental theorem due to Gindikin [Russian Math. Surveys, 29 (1964), 1-89] says that the generalized power  $\Delta_s(-\theta^{-1})$  defined on a symmetric cone is the Laplace transform of a positive measure  $R_s$  if and only if  $s$  is in a given subset  $\Xi$  of  $\mathbb{R}^r$ , where  $r$  is the rank of the cone. When  $s$  is in a well defined part of  $\Xi$ , the measure  $R_s$  is absolutely continuous with respect to Lebesgue measure and has a known expression. For the other elements  $s$  of  $\Xi$ , the measure  $R_s$  is concentrated on the boundary of the cone and it has never been explicitly determined. The aim of the present paper is to give an explicit description of the measure  $R_s$  for all  $s$  in  $\Xi$ . The work is motivated by the importance of these measures in probability theory and in statistics since they represent a generalization of the class of measures generating the famous Wishart probability distributions.

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Date: Received: Jun. 21, 2017; Accepted: Sep. 12, 2017.

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2010 *Mathematics Subject Classification*. Primary 46G12; Secondary 28A25.

*Key words and phrases*. Jordan algebra, symmetric cone, generalized power, Laplace transform, Riesz measure.

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